Building Problem Solving Strategies

Giving Tuesday

by Leanne Luttrell

This document includes 4 problems with detailed solutions.

Teachers and parents of students in grades 3 – algebra should be able to find a problem that will be a challenging, respectful task for your students! We hope these are helpful! Please visit our website, Twitter, or Facebook to share your thoughts! We look forward to hearing from you!

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Building
Problem Solving Strategies

by Leanne Luttrell

Do you want to inspire your students to love problem-solving? Have you been searching for a way to improve problem-solving skills but are overwhelmed by the materials and choices available? These problems and solutions were written for Giving Tuesday in December 2020 as a gift to teachers! As a teacher, I know how hard you all work every year, but this year is especially challenging.

Thank you for all you do!

Logical and problem solving are my passion! If these problems are helpful, I hope you will visit www.Amazing-Minds.com! In addition to the materials that are for sale, you will also find a variety of free materials, including a Mind-Bending Monday challenge!

Building Toys

Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. To make production easier, all the wheels are the same size. Holidays are always busy! On Tuesday afternoon, they received a rush order. It had to be completed in 3 hours! The supervisor checked inventory to make sure they had the 97 wheels needed to complete the order. The order had 5 more cars than motorcycles.

How many cars were in the rush order?

How many motorcycles were in the rush order?

Can you find a strategy that would easily work with greater numbers? Explain your reasoning.

Building Toys: Monday Morning

Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. To make production easier, all the wheels are the same size. Holidays are always busy! On Tuesday afternoon, they received a rush order. It had to be completed in 3 hours! The supervisor checked inventory to make sure they had the 54 wheels needed to complete the order. The order had 3 more cars than motorcycles.
How many cars were in the rush order?
How many motorcycles were in the rush order?

Can you find a strategy that would easily work with greater numbers? Explain your reasoning.

Monthly Production

Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. To make production easier, all the wheels are the same size. One week, they had 1,800 wheels in inventory. That week, the company paid 5.3 hours of overtime and built 54 more cars than 3-wheel motorcycles. At the end of the week, there were 261 wheels remaining in inventory.

How many cars did Toddler Toys produce that week?
How many 3-wheel motorcycles did Toddler Toys produce that week?.

Introducing: Bicycles!

Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. In June, they began building bicycles (2 wheels), also. To make production easier, all the wheels are the same size.

Because Toddler Toys was producing a new product, they produced the same number of bicycles, motorcycles, and cars the first day of the week, then built additional products based on orders. During the first week, there were 59 fewer bicycles ordered than cars. There were 78 more cars ordered than 3-wheel motorcycles. The company used 1817 wheels that week.

How many bicycles did Toddler Toys produce that week?

Building Problem Solving Strategies
Giving Tuesday: Solutions

Note to teachers:
There are challenges in teaching problem solving. I believe one of the biggest challenges is how to react when students are 'stuck'. What kinds of questions do you ask? How can you scaffold with questions without telling them the answer or giving a 'hint' that directly leads them to a solution?

Questioning is one of the most important aspects of teaching. If you are interested in learning more about questioning strategies, I hope you will visit amazing-minds.com to read my blog, view a video, or participate in an online discussion.

The following pages include pictures, tables, equations, etc. to show how your students might have solved the problem. This is not step-by-step for students; it is designed for teacher use and includes additional notes on the problem. *As you know, problems can be solved in multiple ways; this just illustrates a few of those ways!*

Also, keep in mind that an appropriate strategy depends on the age and previous knowledge of the students! Please share your experiences through email, comments on our website, Twitter, and/or Facebook! We look forward to hearing from you!

I hope this is helpful! Thank you!
Leanne

Building Problem Solving Strategies

Building Toys: Possible Solution Strategies

Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. To make production easier, all the wheels are the same size. Holidays are always busy! On Tuesday afternoon, they received a rush order. It had to be completed in 3 hours! The supervisor checked inventory to make sure they had the 97 wheels needed to complete the order. The order had 5 more cars than motorcycles. How many cars were in the rush order? How many motorcycles were in the rush order?
Can you find a strategy that would easily work with greater numbers? Explain your reasoning.

Some students may begin with a table such as this one:

# of motor-cycles	# of cars	# of motor-cycle wheels	# of car wheels	total # of wheels
1	6	3	24	27
2	7	6	28	34

This will eventually lead to the correct answer, but it is not a strategy that would help you solve this with any given numbers.

If students solve this with this or a similar strategy, ask them to then solve a similar problem with larger numbers. **Remember the goal is not just to find the solution, but also to develop strategies that will help solve more challenging tasks!**

Building Cars: Monday Morning is a more accessible problem if students need to build a concrete model. Use the same strategies. The solution: 6 motorcycles and 9 cars. $(6\times3)+(9\times4)=54$

Hopefully, students will represent the '5 more cars than motorcycles', by representing 5 cars.
Look for how students begin. Those who immediately begin with the five cars have a different level of understanding than those who begin with 1 and 6, 2 and 7, then eventually see the pattern.

Depending on their level of problem solving, students may approach this differently. Seeing various representations in your discussion will help all students!
Let's consider these strategies:

Concrete
At a beginning level, students as young as third grade can solve this by using manipulatives or drawing pictures. The larger number encourages finding a pattern, but you can also scaffold for struggling students by solving a simpler problem.

Representational
Students at this level are still using drawings, but those drawings use representations.

Middle school students can then use the representation to create a system of equations.

Abstract
If students are easily able to solve this using a system of equations ***and*** are able to either label or explain their equations, ***then those students should be working on a more challenging problem.***
This would not be a respectful task for them!

Building Problem Solving Strategies

Building Toys: Possible Solution Strategies

Building Toys (page 2)

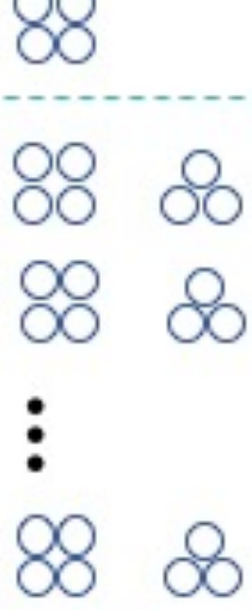
Concrete/Representational

Hopefully, students will represent the '5 more cars than motorcycles' by drawing (or building) 5 cars. Look for how students begin. (See note on table.) Students who are building might use counters to represent wheels. If students try to draw or build the entire item, you might want to do a gallery walk and discuss what is needed to represent them. Hopefully, they will see 4 circles represents a car and 3 circles represents a 3-wheel motorcycle.

The '5 more cars' will always use 20 of the wheels.

$$97 - 20 = 77$$

total wheels	wheels on 5 cars	wheels on sets of cars and motorcycles



As more cars and motorcycles are built, cars and motorcycles are added in pairs so there will always be 5 more cars. There are 7 wheels in each set of 1 car and 1 motorcycle.

If students do not see this, we want to question so they discover this. As they are drawing, ask them to show you how the drawing represents the information in the problem. They can show you '5 more cars'. As they add to the drawing, ask them to show you again. You can also ask them to reflect on what they are having to add each time more wheels are needed.

***After** students find the relationship, ask:*

If there are 77 wheels in this section, how many 'pairs' or sets of cars and motorcycles did Toddler Toys build?

$$77 \div 7 = 11$$

total wheels on sets of cars and motorcycles	wheels on one set of a car and motor-cycle	# of sets of cars and motor-cycles

Building Problem Solving Strategies

Building Toys: Possible Solution Strategies

Building Toys (page 3)

Concrete/Representational

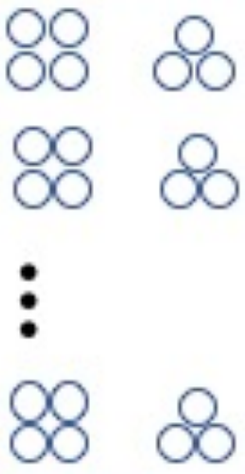
Hopefully, students will represent the '5 more cars than motorcycles' by drawing (or building) 5 cars. Look for how students begin. (See note on table.) Students who are building might use counters to represent wheels. If students try to draw or build the entire item, you might want to do a gallery walk and discuss what is needed to represent them. Hopefully, they will see 4 circles represents a car and 3 circles represents a 3-wheel motorcycle.

The '5 more cars' will always use 20 of the wheels.

5 cars

$$97 - 20 = 77$$

total wheels	wheels on 5 cars	wheels on sets of cars and motorcycles



11 cars

11 motorcycles

$$77 \div 7 = 11$$

total wheels on sets of cars and motorcycles	wheels on one set of a car and motor-cycle	# of sets of cars and motor-cycles

Finally, students can use this to determine the total number of each item.

The rush order was for 16 cars and 11 motorcycles.

Students can check their work using equations. They should use an equation for each piece of information.

# of items		
$11 + 5 = 16$		
# of motor-cycles	*5 more*	# of cars

# of wheels		
$33 + 64 = 97$		
# of wheels on motor-cycles	# of wheels on cars	total wheels

Notice that having elementary students check their work in this way not only shows reasoning, but it also builds a conceptual foundation for systems of equations!

Building Toys: Possible Solution Strategies

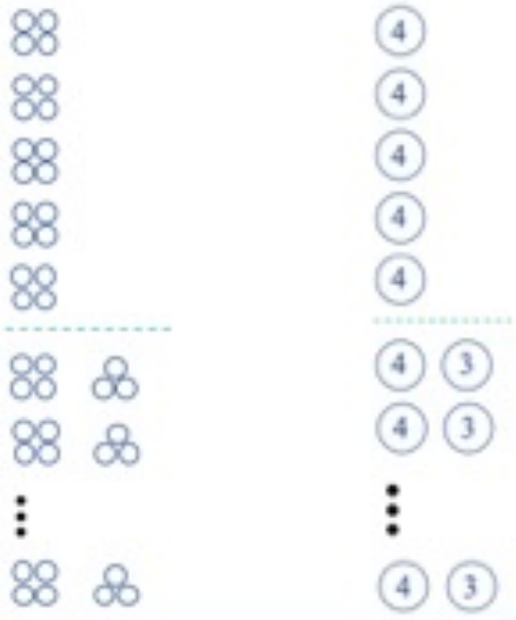
Representational and Abstract

Building Toys (page 4)

There are multiple ways to solve problems! Here are some examples.

Representational

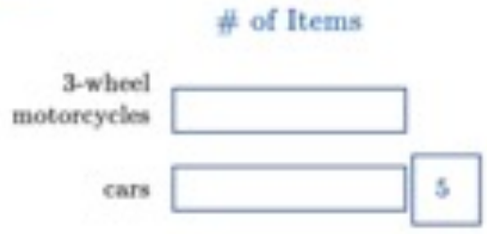
Some students will understand and create representations using numbers rather than building or drawing individual circles to represent wheels. For example:



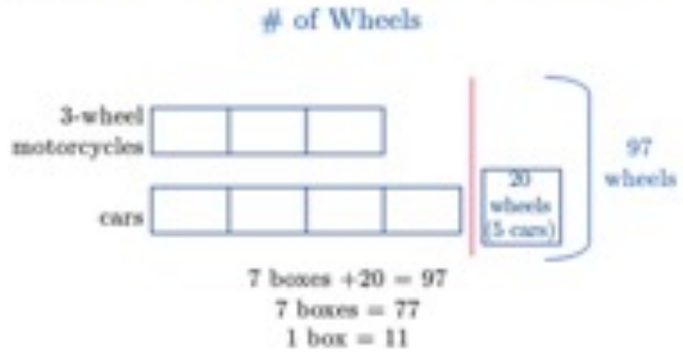
The reasoning and equations would be the same as described on the previous page.

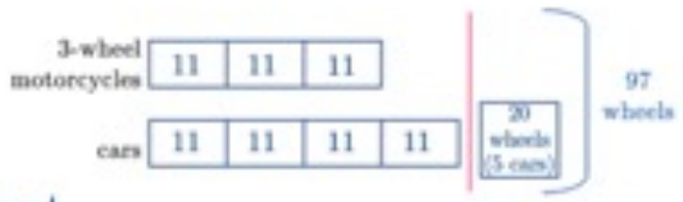
Representational

Another way students might represent this is using Singapore Method of Problem Solving, or Bar Models. This is unlikely for elementary students, because students would have to use the number of wheels rather than items, so students would first use the bar models to represent ratios of wheels to find the total number of wheels on each product. ***This is a more advanced application of this strategy. If your middle school students don't know this strategy, it would be a great time to introduce it!***



*Note that this model still requires a conceptual understanding of the visual model to the left.





Abstract

Students at this level are able to independently define the variables, write the system of equations, **and explain their reasoning**.

	# of items	# of wheels
x = # of 3-wheel motorcycles		
y = # of cars	$x + 5 = y$	$3x + 4y = 97$

Students can then use substitution, elimination, or graphing to solve. You could have a discussion about which would be most efficient. (I would not use graphing for this one!)

Building Problem Solving Strategies

Monthly Production: Possible Solution Strategies

Representational

It is unrealistic to use concrete models for this problem. If students need more scaffolding, have them solve a simpler problem so they can find and apply patterns and strategies.

Students obviously won't draw every 'car', but this type of drawing helps students visualize the problem. For questioning ideas, see the notes on 'Building Toys'.

(4) ⋮ (4) 54 cars

The '54 more cars' will always use 216 of the wheels. 54×4=216

Total Wheels Used:
1,800 – 261 = 1,539

(4) (3) ⋮ (4) (3)

As more cars and motorcycles are built, cars and motorcycles are added in 'pairs' so there will always be 54 more cars. There are 7 wheels in each set of 1 car and 1 motorcycle.

$$1{,}539 - 216 = 1{,}323$$

total wheels | wheels on 54 cars | wheels on sets of cars and motorcycles

$$1{,}323 \div 7 = 189$$

total wheels on sets of cars and motorcycles | wheels on one set of a car and motorcycle | # of sets of cars and motorcycles

189 cars and 189 motorcycles

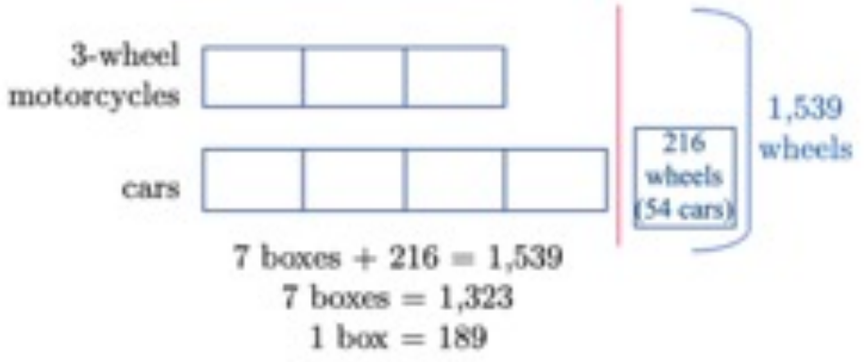
Another way students might represent this is using Singapore Method of Problem Solving, or Bar Models. ***This is a more advanced application of this strategy. If your middle school students don't know this strategy, it would be a great time to introduce it!***

# of Items

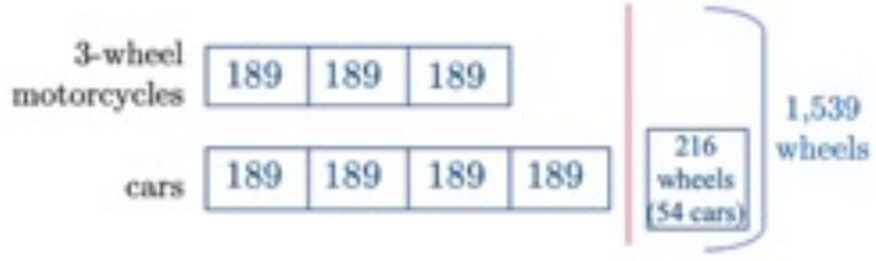
3-wheel motorcycles: ☐

cars: ☐ 54

# of Wheels



7 boxes + 216 = 1,539
7 boxes = 1,323
1 box = 189



Toddler Toys produced 189 motorcycles and 243 cars that week.

Building Problem Solving Strategies

Monthly Production: Possible Solution Strategies

Monthly Production (page 2)

Abstract

Students at this level are able to independently define the variables, write the system of equations, **and explain their reasoning**.

First, students should recognize the overtime hours are not relevant. To find the number of wheels used that month, subtract the remaining wheels from the original inventory.

Total Wheels Used:
1,800 − 261 = 1,539

Students may assign the variables in different ways (x as the number of cars, m for motorcycles, etc.), but it is important they recognize the variables represent the number of items, not the number of wheels. If, in the definition, students just write 'cars', ask them to explain in more detail.

x = # of 3-wheel motorcycles
y = # of cars

Students can label their equations. All students should be able to explain each term. (The term 4y represents 4 wheels on each car.) If students are struggling, labeling each term will help them see their mistakes and misunderstandings.

# of items	# of wheels
$x + 54 = y$	$3x + 4y = 1{,}539$

Students can then use substitution, elimination, or graphing to find the solution. You could have a discussion about which would be most efficient. (I would not use graphing for this one!)

$$3x + 4(x + 54) = 1{,}539$$
$$3x + 4x + 216 = 1{,}539$$
$$7x + 216 = 1{,}539$$
$$7x = 1{,}323$$
$$x = 189$$

Although it is not shown, students should be expected to show the strategy they used, determine if the solution is reasonable, and check their work by using the solutions to evaluate the equations in the system.

Toddler Toys produced 189 motorcycles and 243 cars that week.

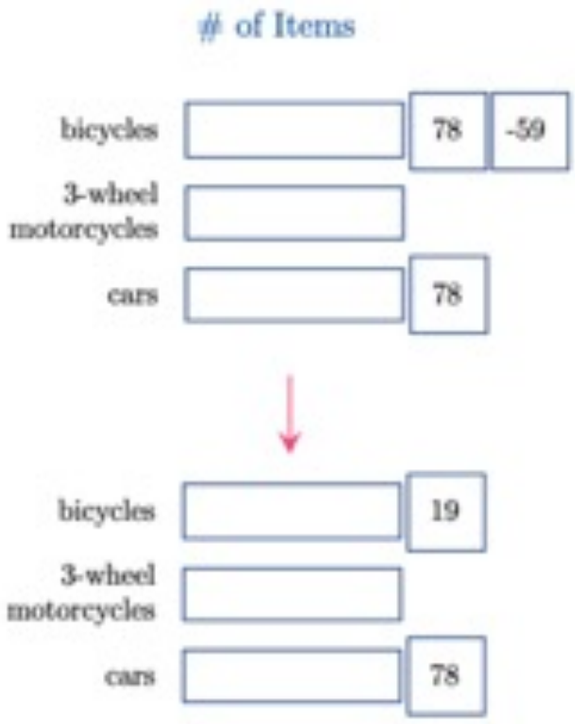
Building Problem Solving Strategies

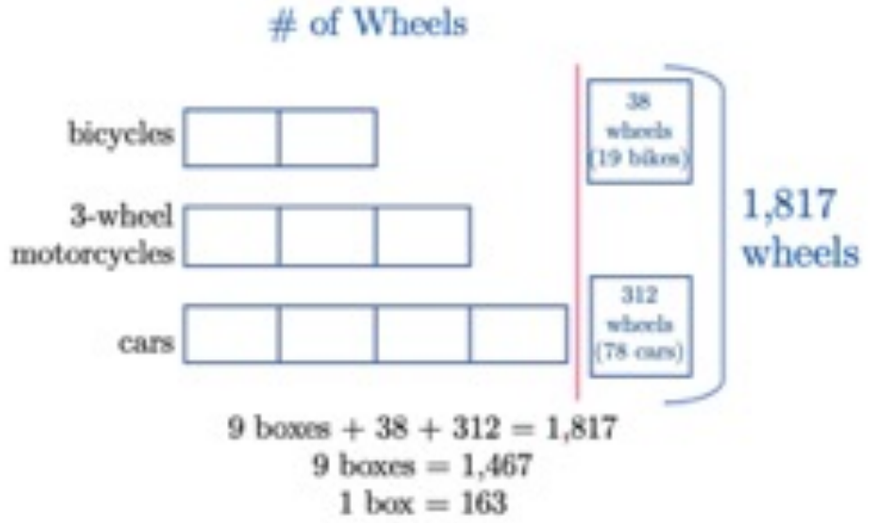
Introducing: Bicycles! : Possible Solution Strategies

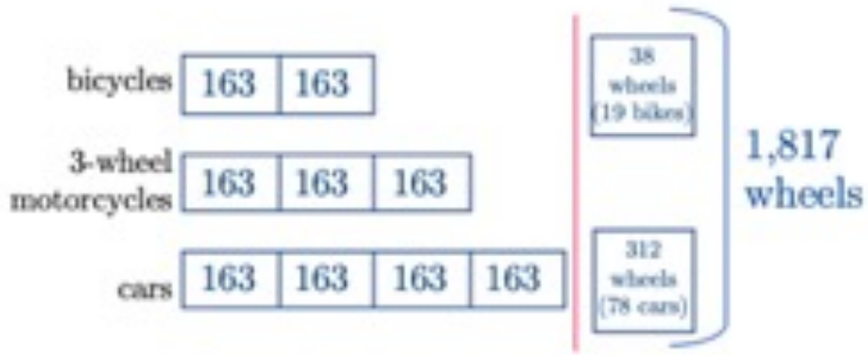
Toddler Toys builds 3-wheel motorcycles and 4-wheel cars for toddlers. In June, they began building bicycles (2 wheels), also. To make production easier, all the wheels are the same size. Because Toddler Toys was producing a new product, they produced the same number of bicycles, motorcycles, and cars the first day of the week, then built additional products based on orders. During the first week, there were 59 fewer bicycles ordered than cars. There were 78 more cars ordered than 3-wheel motorcycles. The company used 1817 wheels that week. How many bicycles did Toddler Toys produce that week?

Representational

One way students might represent this is using Singapore Method of Problem Solving, or Bar Models. To do this, students will need to use an algebraic model to find the relationship between products, then use a SMPS ratio model to find the number of wheels. ***This is a more advanced application of this strategy. If your middle school students don't know this strategy, it would be a great time to introduce it!***







This strategy is so helpful for many students! This document, however, does not explain the strategy.

For more support, email Leanne@Amazing-Minds.com with questions.

Toddler Toys produced 182 bicycles, 163 motorcycles, and 241 cars that week.

Introducing: Bicycles! : Possible Solution Strategies

Introducing Bicycles! (page 2)

Abstract

This will be challenging to solve abstractly for students in middle school. It is important that students be able to **explain their reasoning**. Instead of using three variables, here is one strategy advanced students at this age might use.

There is more than one way to represent this algebraically. Students can use 3 variables, but in middle school, many students will write equations in terms of one variable. This will be much easier for students who have a background in SMPS. Students with that experience can relate the variable to the item with a single 'bar' in a bar model.

$$\begin{aligned} \text{\# of bicycles} &= x + 78 - 59 = x + 19 \\ \text{\# of 3-wheel motorcycles} &= x \\ \text{\# of cars} &= x + 78 \end{aligned}$$

The number of items is not given, so the equation represents the number of wheels. Students should explain the equation.

\# of wheels

$$2(x + 19) + 3x + 4(x + 78) = 1{,}817$$

$$2x + 38 + 3x + 4x + 312 = 1{,}817$$

$$9x + 350 = 1{,}817$$

$$9x = 1{,}467$$

$$x = 163$$

Once students find the value of x, they can easily evaluate each of the equations above to find the number of each item that was produced that week.

Students should also be expected to determine if the solution is reasonable.

Toddler Toys produced 182 bicycles, 163 motorcycles and 241 cars that week.